Sections 5.1 & 5.2 – I.C.E – Trig Identities

xSimplify the following to ONE trig function or numerical value:

1) $\cos^2\beta+\cos^2\left(\frac{\pi}{2}-\beta\right)$

$=\cos^2\beta+\sin^2\beta$

$=1$

2) $\sin t\csc\left(\frac{\pi}{2}-t\right)$

$=\sin t\sec t$

$=\sin t\cdot\frac{1}{\cos t}$ $=\tan t$

3) $\dfrac{\cos\left(\frac{\pi}{2}-x\right)}{\sin\left(\frac{\pi}{2}-x\right)}$

$=\dfrac{\cos x}{\sin x}=\cot x$

4) $\sec y\cos y$

$=\frac{1}{\cos y}\cdot\frac{\cos y}{1}=1$

5) $(1+\sin y)(1+\sin(-y))$

$(1+\sin y)(1-\sin y)$

$=1-\sin^2 y$

$=\cos^2 y$

6) $\sec^2\left(\frac{\pi}{2}-x\right)-1$

$=\csc^2 x-1$

$=\cot^2 x$

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Prove the following identities: be sure to only work ONE side of the equation!

7) $\frac{1}{\tan x}+\frac{1}{\cot x}=\tan x+\cot x$

$= \frac{\cot x+\tan x}{\tan x \cot x} = \frac{\cot x+\tan x}{\tan x\left(\frac{1}{\tan x}\right)} = \cot x+\tan x$ ✓

8) $\cot\alpha+\tan\alpha=\csc\alpha\sec\alpha$

$\frac{\cos\alpha}{\sin\alpha}+\frac{\sin\alpha}{\cos\alpha} = \frac{\cos^2\alpha+\sin^2\alpha}{\sin\alpha\cos\alpha} = \frac{1}{\sin\alpha\cos\alpha}$

$= \csc\alpha\sec\alpha$ ✓

9) $\sin^2\alpha-\sin^4\alpha=\cos^2\alpha-\cos^4\alpha$

$\sin^2\alpha(1-\sin^2\alpha) = (1-\cos^2\alpha)(\cos^2\alpha) = \cos^2\alpha-\cos^4\alpha$ ✓

10) $\frac{\cot^3 t}{\csc t}=\cos t(\csc^2 t-1)$

$(\cot^2 t)(\cot t)(\sin t) = (\csc^2 t-1)\left(\frac{\cos t}{\sin t}\right)(\sin t)$

$= \cos t(\csc^2-1)$ ✓

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11) $\frac{1+\sin\theta}{\cos\theta}+\frac{\cos\theta}{1+\sin\theta}=2\sec\theta$

$= \frac{(1+\sin\theta)^2+\cos^2\theta}{\cos\theta(1+\sin\theta)} = \frac{(1+2\sin\theta+\sin^2\theta)+\cos^2\theta}{\cos\theta(1+\sin\theta)}$

$= \frac{2+2\sin\theta}{\cos\theta(1+\sin\theta)} = \frac{2(1+\sin\theta)}{\cos\theta(1+\sin\theta)} = 2\sec\theta$ ✓

12) $\tan^2\theta+4=\sec^2\theta+3$

$= (\sec^2\theta-1)+4 = \sec^2\theta+3$ ✓

13) $\cot^2 y(\sec^2 y-1)=1$

$\cot^2 y(\tan^2 y) = \frac{1}{\tan^2 y}\cdot\frac{\tan^2 y}{1} = 1$ ✓

14) $\dfrac{1}{\sec x \tan x} = \csc x - \sin x$

$= \cos x \cdot \cot x = \cos x\left(\dfrac{\cos x}{\sin x}\right) = \dfrac{\cos^2 x}{\sin x}$

$= \dfrac{1-\sin^2 x}{\sin x} = \dfrac{1}{\sin x} - \dfrac{\sin^2 x}{\sin x}$

$= \csc x - \sin x$ ✓

Here's a tough one if you want a challenge!

15) $\dfrac{\tan x + \tan y}{1 - \tan x \tan y} = \dfrac{\sin x \cos y + \cos x \sin y}{\cos x \cos y - \sin x \sin y}$

$\dfrac{\frac{\sin x}{\cos x} + \frac{\sin y}{\cos y}}{1 - \frac{\sin x}{\cos x} \cdot \frac{\sin y}{\cos y}} = \dfrac{\frac{\sin x \cos y + \cos x \sin y}{\cos x \cos y}}{\frac{\cos x \cos y - \sin x \sin y}{\cos x \cos y}}$

$= \dfrac{\sin x \cos y + \cos x \sin y}{\cos x \cos y} \cdot \dfrac{\cos x \cos y}{\cos x \cos y - \sin x \sin y}$

$= \dfrac{\sin x \cos y + \cos x \sin y}{\cos x \cos y - \sin x \sin y}$ ✓